Stability and bifurcations in nonlinear discrete-time models of competition between two age-stage structured plant populations

Nikolay N. Zavalishin[1*], Nikolay N. Zavalishin[1], Nina G. Ulanova[2], Dmitrii O. Logofet[3]

[1] Laboratory of mathematical ecology, A.M. Obukhov Institute of atmospheric physics RAS
[2] Dept. of Geobotany, Biological Faculty, Lomonosov Moscow State University
[3] Laboratory of mathematical ecology, A.M. Obukhov Institute of atmospheric physics RAS
[*] Electronic Address: `nickolos@ifaran.ru; nickolos@pkbct.ru`

Populations of raspberry (Rubus idaeus) and aspen (Populus tremula) often occupying clear-cut forest areas can be described by a set of age-stage groups of individuals differing both in the ontogenetic stage and chronological age. For each population a linear discrete time matrix model can be designed and calibrated on the basis of two reproductive phases representation: seed-origin and vegetative. During first years after felling shrubs of the seed-origin type determines the population dynamics which demonstrates an exponential growth reflected by the linear model. Further succession is connected mainly with a vegetative type, and such strong factors as competition for natural resources with other species start play a leading role that can be simulated only by means of a nonlinear model. Generally, if possible competitive relations between a variety of age-stage groups for both populations were reliably established, a kind of multi-dimensional competition model could be constructed and studied. Due to the lack of information on these relations, functional forms of group interactions are hardly to be selected adequately from a biological point of view. Therefore, strongly aggregated minimal model available for mathematical analysis is designed on the basis of modifying the linear matrix model by multiplying each component of its equations on the nonlinear term reflecting the competition pressure of aggregated groups from one population on the corresponding group of another. In this aggregation we distinguish only two functional groups for one population (aspen) and three for the other obtaining five-dimensional nonlinear discrete-time matrix model. Asymtotically four of them remain essential and give eight steady states corresponding to different competition outcomes. Stability conditions for all equilibria are obtained together with cyclic and chaotic dynamics. Since plant populations are usually inertial, conditions of absence for chaos are considered. Calibration of the aggregated system by real data from field measurements as well as its aggregation to the minimal one initiates

additional mathematical optimization problems.

This work is supported by the grant 05-04-49291 of the Russian Fund of Basic Research.