Analytical calculations of Lyapunov spectra of high-dimensional billiards

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For a better understanding of the chaotic behavior of systems of many moving particles it is useful to look at other systems with many degrees of freedom. The dynamics of a system consisting of many spherical hard particles can be described as a single point particle moving in a high-dimensional space with fixed hyper-cylindrical scatterers with specific orientations and positions. We investigate the similarities in the Lyapunov exponents between systems of many particles and high-dimensional billiards. We calculate the full spectrum of Lyapunov exponents for the high-dimensional dilute random Lorentz gas in an arbitrary number of dimensions [1]. We find that the spectrum becomes flatter with increasing dimensionality. These results are relates to the shape of the scatterers. We therefore investigate billiards with cylindrical scatterers which have isotropically distributed orientations and homogeneously distributed positions [2]. Numerical results are compared to numerical results for systems of many hard particles as well as the analytical results for the high-dimensional Lorentz gas. The smaller positive exponents of the billiard with cylindrical scatterers behaves much more like the exponents of hard-disk systems than the exponents of the Lorentz gas. This suggests that the hard-disk systems may be approximated by a spatially homogeneous and isotropic system of scatterers for a calculation of the smaller Lyapunov exponents. The method of the partial stretching factor, which was used for the Lorentz gas, is used together with a low-density approximation [3] to calculate these exponents analytically, with results that compare well with simulation results of hard disks and hard spheres, both qualitatively and quantitatively.

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