Combined effects of frequency quantisation and additive input noise in a first-order digital PLL

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A recent work by Gardner [1] on the subject of digital phase-locked loops (DPLLs) investigated, via simulation, the characteristics of the phase-jitter caused by frequency quantisation in the numerically-controlled oscillator. Further works by Feely, Teplinsky et al [2, 3] used the theory of nonlinear dynamics to provide a complete analytical explanation of this phase- jitter.

This paper examines in detail the case where the input signal is embedded in additive noise, a scenario earlier investigated by Gardner where no satisfactory method of characterising the phase- jitter was found. Here, further numerical results for the 1-D DPLL are presented and it is shown analytically how the DPLL noise-jitter dynamics may be approximated by a noisy circle rotation map for reasonable levels of additive noise. The noise in this case is unique and highly nonlinear in nature and thus not amenable to traditional analysis. By considering the the probability flow over time, a time- dependent difference-delay equation is derived for the probability density function (PDF) of the phase-jitter. It is shown that this PDF reaches a steady-state and that this state is described by a non- local equation. The solutions of this equation are investigated, both numerically and analytically, and used to explain the interaction between the additive and quantisation noise that was previously not understood.

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[1] F. M. Gardner,"Frequency granularity in digital phase-locked loops", IEEE Trans. Commun., vol. 44, pp. 749 V758, June 1996.

[2] A. Teplinsky, O. Feely and A. Rogers,"Phase-jitter dynamics of digital phase-locked loops", IEEE Trans. Circuits and Systems, Part I: Fundamental Theory and Applications, 46, pp. 545-558, May 1999.

[3] A. Teplinsky and O. Feely,"Phase-jitter dynamics of digital phase- locked loops: Part II", IEEE Trans. Circuits and Systems, Part I: Fundamental Theory and Applications., 47, pp. 458-473, April 2000.