

## Bifurcations in a family of supply chain models

Jan Topi Tervo<sup>1\*</sup>, Michael Freitag<sup>1</sup>, Bernd Scholz-Reiter<sup>1</sup>, Karsten Peters<sup>2</sup>

<sup>1</sup> Bremen Institute of Industrial Technology and Applied Work Science (BIBA)

Hochschulring 20, 28359 Bremen, Germany

<sup>2</sup> TU Dresden, Institute for Traffic and Economics

Andreas-Schubert Str. 23, 01062 Dresden, Germany

\* Electronic Address: [ter@biba.uni-bremen.de](mailto:ter@biba.uni-bremen.de)

In the past decade, physicists have become more and more interested in interdisciplinary applications of dynamical systems theory in fields such as biophysics, sociophysics or econophysics. However, the dynamic behaviour of production and logistics networks has only recently raised an increasing interest in the scientific community [1, 2, 3].

Considering discontinuities in the processes and a generally non-synchronous flow of material and information in network-like systems, modelling and understanding of the dynamics of these complex systems is challenging. Several previous works uncovered a variety of oscillatory and even chaotic behaviour in production systems and supply chains [1, 4, 5]. Whereas traditional modelling approaches focussed on stochastic queueing models, the recent formulation of continuous flow models for the information and material flows in supply networks led to new insights. Helbing et al. [3] found both, convective and absolute instabilities in continuous flow models of supply chains, which induce in certain parameter regions increasing oscillations along a supply chain. These observations may explain the so called bull-whip or Forrester-effect from first principles.

While the above mentioned results for supply networks were obtained on linearized dynamic equations, we focus on the effects of nonlinearities in supply chains, caused by shortages in stocks and nonlinear relations in the adaptation of production rates towards market demands. It turns out, that these nonlinearities give rise to a rich variety of bifurcations. We investigate these bifurcation scenarios and the related critical parameters in terms of workload, adaptation speeds and also different rules which can be interpreted as order policies.

The results provide explanations for period doublings which have been obtained in supply chain data and may be used to optimize the structure of the material- and information flow in order to obtain more reliable, stable and robust supply networks.

- 
- [1] E. R. Larsen, J. D. W. Morecroft, J. S. Thomsen, Complex behaviour in a production-distribution model, *Europ. J. Op. Res.* 119 (1999) 61–74.
  - [2] C. F. Daganzo: *A Theory of Supply Chains*, LNEMS, Springer, New York 2003
  - [3] D. Helbing: Production, supply, and traffic systems: A unified description. In: S. P. Hoogendoorn, S. Luding, P. H. L. Bovy, M. Schreckenberg, and D. E. Wolf (eds.) *Traffic and Granular Flow '03*, pp. 173-188, Springer, Berlin, 2005.
  - [4] D. Helbing, T. Seidel, S. Lämmer and K. Peters: Self-organization principles in supply networks and production systems, pp. 531-555 in: B.K. Chakrabati et al. (eds.): *Econophysics and Sociophysics: Trends and Perspectives*, Wiley-VCH, 2006
  - [5] K. Peters, J. Worbs, U. Parlitz and H.-P. Wiedahl: Manufacturing systems with restricted buffer sizes, in: *Nonlinear Dynamics of Production Processes*, Edited by G. Radons and R. Neugebauer (Wiley-VCH, Weinheim), 39-54 (2004)