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About the Capture Rate of Small Diffusing Molecules on a Cylindrical Membrane

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Biomembranes consist of a lipid bilayer into which proteins are embedded to fulfill numerous tasks in localized regions of the membrane. Often, the proteins have to reach these regions by simple diffusion. Motivated by the cylindrical topology of the membrane of the endoplasmic reticulum, we calculate the capture rate of small diffusing molecules on a cylindrical surface based on the *Smoluchowski* approach [1]. The problem can be reduced to the solution of the diffusion equation on a finite domain with a small absorbing hole and otherwise reflecting boundaries. The solution is constructed in terms of an asymptotic series starting from appropriate 'inner' and 'outer' solutions. Asymptotic matching yields the decay rate of the exponentially decaying capture rate and fixes its magnitude. Our main result is

$$k(t) = |\Omega_0|\,\lambda e^{-\lambda t}$$

with

$$\lambda = \frac{2\pi D}{|\Omega_0|}\frac{1}{\log\frac{1}{\delta}}\left(1 - \frac{2\pi}{\log\frac{1}{\delta}}R(0;0)\right)$$

where $|\Omega_0|$ is the area of the cylindrical surface, δ is the radius of the diffusing molecules, D is the diffusion constant and $R(0;0)$ is the regular part of an associated *Green*'s function which can be calculated analytically. The so obtained capture rate $k(t)$ can be regarded as an upper bound of diffusion limited reactions on a cylindrical membrane.

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[1] M. von Smoluchowski, *Z. Phys. Chem.* **92**, 129 (1917).