

**The reinfection threshold exists  
... and is the mean field version of the transition between  
annular and compact growth in spatial models**

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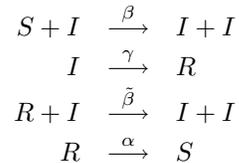
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Recently, the notion of a reinfection threshold in epidemiological models of only partial immunity has been debated in the biological literature [1, 2, 3]. We present a rigorous analysis of a model of reinfection which shows a clear threshold behaviour at the parameter point where the reinfection threshold was originally described.

Furthermore, we demonstrate that this threshold is the mean field version of a transition in corresponding spatial models of partial immunization. The reinfection threshold corresponds to the transition between annular growth of an epidemics spreading into a susceptible area leaving recovered behind and compact growth of a susceptible-infected-susceptible region growing into a susceptible area. This transition between annular growth and compact growth was described in the physics literature [4] long before the reinfection threshold debate broke out in the theoretical biology literature and is still topic of ongoing research, see e.g. [5].

The basic model which we consider is of SIRI type



with  $S$  susceptible,  $I$  infected,  $R$  recovered hosts. Start of the analysis is a spatial master equation of the above given reaction scheme.

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