# Inertia-induced coherent structures in time-periodic viscous flows with one invariant

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Three-dimensional advection of passive tracers in time-periodic viscous flows serves as model problem for laminar mixing in industry. Such flows admit analysis in terms of volume-preserving maps classified by the number of invariants ('actions') [1]. An important aspect for mixing applications is the response of coherent structures that occur in one-action (invariant surfaces) and two-action (invariant curves) maps to inertial perturbations. These coherent structures form barriers to tracer transport and their destruction (by inertial effects) is an essential condition for efficient mixing. Two-action maps and one-action maps with invariant tori have been studied extensively in this respect [1]. One-action maps with invariant surfaces other than tori have not. This motivates our numerical study on the topological changes undergone by an experimentally-realisable one-action map, with invariant surfaces topologically equivalent to spheres, from its non-inertial baseline.

Considered is the time-periodic flow set up in the non-dimensional square cylinder $\mathcal{D} : [r, \theta, z] = [0, 1] \times [0, 2\pi] \times [-1, 1]$ by a two-step forcing protocol. The first and second forcing steps involve steady translation of the bottom wall ($z = -1$) in positive $x$ and $y$-direction, respectively, with fixed non-dimensional displacement $D = VT/2L = 5$ ($V$: translation velocity; $T$: period time; $L$: side length). The time-periodic flow $\boldsymbol{v}(\boldsymbol{x}, t)$ on symmetry grounds admits expression entirely in terms of the steady flow $\boldsymbol{u}(\boldsymbol{x})$ during the first forcing step: $\boldsymbol{v}(\boldsymbol{x}, t) = \boldsymbol{u}(\boldsymbol{x})$ (first step); $\boldsymbol{v}(x, y, z, t) = \boldsymbol{u}(y, -x, z)$ (second step). The steady flow $\boldsymbol{u}(\boldsymbol{x})$ is governed by the non-dimensional steady Navier-Stokes equations with the Reynolds number $Re = VL/\nu$ ($\nu$: kinematic viscosity) as single control parameter. High-precision resolution of $\boldsymbol{u}(\boldsymbol{x})$ is attained with a specially-developed spectral scheme [2].

The non-inertial baseline ($Re = 0$) of the time-periodic flow $\boldsymbol{v}$ possesses one invariant and relates to a one-action map with invariant surfaces that are topologically equivalent to spheres rather than the common case of tori [3]. This has fundamental ramifications for the response to 'small' departures from the non-inertial limit and leads to a new type of response scenario: resonance-induced merger of coherent structures. Thus several coexisting families of two-dimensional coherent structures are formed that make up two classes: fully-closed structures and leaky structures. Fully-closed structures restrict motion as in a one-action map; leaky structures have open boundaries that connect with a locally-chaotic region through which exchange of material with other leaky structures occurs. For 'large' departures from the non-inertial limit, the above structures vanish and the topology becomes determined by isolated periodic points and associated manifolds. This results in unrestricted and truly three-dimensional chaotic advection.

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