Direct or indirect: Partial phase synchronization for multivariate synchronizing systems

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The analysis of dynamical systems usually relies on observations of more than one single process. Of particular interest is the detection of interactions between processes in such multivariate systems. When more than two processes are analyzed, one has to face the problem that complex interaction structures between the processes may arise. For example, it is not necessarily the case that two processes in a multivariate system have to interact directly. Bivariate analysis is often not sufficient to reveal the correct interaction structure, i.e. distinguishing between direct and indirect interactions.

Graphical models applying partial coherence have been introduced to discriminate between spurious and non-spurious interactions in multivariate *linear* systems [1]. Especially for weakly coupled, self-sustained, possibly chaotic oscillators phenomena have been observed that cannot be revealed by partial coherence analysis. These oscillators can synchronize their phases, which can be detected by phase synchronization analysis. If more than two oscillators are involved, the problem arises that it is hardly possible to decide by bivariate analysis whether the two oscillators are directly coupled or whether the coupling is mediated by other oscillators.

A methodology is desired similar to graphical models applying partial coherence to *linear* systems that is able to distinguish between direct and indirect coupling in *non-linear phase synchronizing* systems. To this aim, we present the concept of partial phase synchronization, i.e. we study phase synchronization in multivariate systems. We propose a method which can distinguish direct phase synchronization of two components and phase synchronization that is mediated by third components. In particular, the results from linear partial coherence analysis are carried over to the mean phase coherence [2]. We demonstrate its ability to discriminate direct from indirect interactions in a multivariate system of weakly coupled, self-sustained, chaotic oscillators.

[1] R. Dahlhaus, *Metrika* **51**, 157 (2000).

[2] B. Schelter et al., *Phys Rev Lett* **96**, 208103 (2006).