

Co-dimension 2,3 and 4 discontinuity induced bifurcations and their interaction

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Piecewise-smooth dynamical systems with discontinuous system function were intensively investigated in the last years, motivated among other reasons by many practical applications as well as by Poincaré return maps of dynamical systems showing chaotic dynamics. However, bifurcations with co-dimension $n > 1$ occurring in piecewise-smooth systems are still far away from being understood completely.

In this work we investigate the properties of a special type of co-dimension three bifurcation, which we have detected in several maps with discontinuous system function. In the systems we consider, bifurcations with co-dimension one are caused by simple border collisions. Looking at the 2D parameter spaces, we detect some co-dimension two bifurcations points, where an infinite number of co-dimension one bifurcation curves intersect. As a next step, we consider 3D parameter spaces, and identify here the points, where an infinite number of co-dimension two bifurcation curves intersect. These co-dimension three bifurcations, reported by first time in [1], serve as organizing centers for periodic and quasi-periodic dynamics. Unfolding these bifurcations, we are able to explain several strange looking bifurcation structures, which are often observed in 1D and 2D parameter subspaces and which are really difficult to understand otherwise.

Further we investigate the boundaries of the influence areas of the detected co-dimension three bifurcations. Such an influence area is defined as the union of the stability areas of all periodic and quasi-periodic dynamics emerging at the co-dimension three bifurcation point. Based on the structure of this boundary we are able to explain and to classify transitions to chaos, which occur at this boundary. Until now these transitions were reported in many publications, but not explained systematically. Finally, tracking the paths of the co-dimension three bifurcations in the 4D parameter space for one of the investigated models, we demonstrate, that the overall structure of this 4D parameter space is dominated by a single discontinuity induced co-dimension four bifurcation.

The theoretical part of the work is illustrated by several applications examples from the field of electronic circuits. Especially, some models of DC/DC converters [2] and Σ/Δ modulators [3] are investigated in detail.

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