Detection of multi-band chaotic attractors

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The occurrence of multi-band chaotic attractors (MBCA) represents a well-known phenomenon on the field of nonlinear dynamics. For instance, the period doubling cascade occurring in several dynamical systems is typically followed by an inverse band merging cascade. Whereas the first one is formed by a sequence of periodic attractors with periods $p_0 \cdot 2^n$ with increasing n, the second one represents a sequence of MBCAs with $p_0 \cdot 2^n$ bands, whereby n decreases from infinity to one [1]. This bifurcation scenario is well-known and investigated in detail. However, the question, which other types of bifurcation scenarios can be caused by multi-band chaotic attractors and their interactions with each other and with unstable orbits, is still insufficiently investigated. When dealing with this question, it turns out, that efficient algorithms for investigation of MBCAs are missing. The seemingly simple question, how to determine the number of bands of a MBCA represents in fact a hard task from the numerical point of view.

In this work two methods for the detection of the number of bands of a MBCA are presented. The first one is developed for dynamical systems with a continuous system function. It can be shown, that in this case the bands of a MBCA are visited by an orbit in the same order for all times. This fact combined with the mixing property of chaotic attractors allows us to solve the given task efficiently, with low requirements with respect to computation time and memory consumption. The second method does not use any assumptions concerning the properties of the system function and hence can be applied to dynamical systems with discontinuous system function as well. The price for this is the lower convergence rate and the higher memory consumption.

For both methods we present the results of the performance tests, as well as some techniques for ajustment of the parameters. Both methods are validated by the investigation of scenarios in several well-known dynamical systems.

Using our second method we investigate some models of electronic cirquits of practical interest (DC/DC converters [2] and Σ/Δ modulators [3]). These models are intensively investigated in the last few years, especially due to the increasing interest on dynamical systems with a discontinuous system function. Hereby we present and explain a number of complex bifurcation scenarios formed by MBCAs, not reported yet.

The developed methods are implemented within the AnT *4.669* package for simulation and investigation of dynamical systems [4], which is free software and can be downloaded from the website: www.AnT4669.de.

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[3] O. Feely. Nonlinear dynamics of sigma-delta modulation. In *Proc. Midwest Symp. Circuits Syst.*, pages 760-763, 1992.

[4] V. Avrutin and M. Schanz. Ant *4.669* – a tool for simulating and investigating dynamical systems. In E. Schöll and K. Lüdge, editors, *Proc. of XXV Int. Conf. Dynamic Days Europe*, volume 29 E of *Europhysics Conference Series.* TU Berlin, 2005.