Dynamical Singularities in Adaptive Delayed-Feedback Control Systems

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We expand delayed-feedback control (DFC) [1], which is a method for controlling chaos, into adaptive control for discrete time dynamical systems[1]. We then investigate various properties of the adaptive DFC by means of linear stability analysis and numerical simulations using low-dimensional chaotic maps. In particular, we focus on the dynamic characteristics of the adaptive DFC, because systems capable of adaptive behaviors, such as learning, have been reported to often exhibit highly dynamic and complex behavior (e.g., Ref. [2, 3, 4]), in particular, 'neutral' behavior such as that exemplified by zero Lyapunov exponents [5].

As a result, we show that the dynamics of the adaptive DFC systems is indeed neutral in that the finite-time Lyapunov exponent is almost 0 and that transient time distribution decays with a power law. Furthermore, we show that this result can be explained by the following two characteristics of the adaptive DFC systems: (1) the Jacobian matrix of the adaptive DFC systems has eigenvalue, unity at all points in phase space, and (2) parameters approach a stability boundary which is proven to be identical to that of (non-adaptive) DFCs.

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[1]DFC was originally proposed as a method for controlling chaos in continuous dynamical systems [1]. In this case, however, the resulting control systems, described by delay differential equations, are infinite-dimensional, and therefore difficult to analyze. On the other hand, such difficulties do not arise in the case of the discrete time DFC.