On hyperbolic attractors and arithmetic

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1. **The existence and dynamical classification theorem of codimension 1 attractors of the expanding type**. Let A: $T^n \to T^n$ be a hyperbolic automorphism of a torus T^n. Consider a set of couples (A, P) and triples (A, P, Θ), where A is unimodular integral codimension 1 matrix (the unit circle divides its spectrum into two sets, one of which consists of sole real number), P is finite A-invariant set of torus T^n, $\Theta : T^n \to T^n$ is involution.

The couples (A, P) (A_1, P_1) (the triplets (A, P, Θ) (A_1, P_1, Θ_1)) are called equivalent if there is linear mapping Ψ of torus T^n such that

$$\Psi A \Psi^{-1} = A_1, P_1 = \Psi P, \Theta_1 = \Psi \Theta \Psi^{-1}.$$

Theorem A [1]. For every orientable (nonorientable) codimension 1 attractor of the expanding type there is a matching class of attractor consisting of equivalent couples (triplets). Two attractors are topologically conjugate if and only if their classes are identical.

2. **The topological classification theorem of codimension 1 attractors of the expanding type**.

Definition. Let A be an integral unimodular matrix. Define a centralizer $Z(A)$ to be the collection of integral matrices commutative with A.

Theorem B [2]. Orientable (nonorientable) attractors Λ_1, Λ_2 of the classes (A_1, P_1), (A_2, P_2) $((A_1, P_1, \Theta_1), (A_2, P_2, \Theta_2))$ are homeomorphic if and only if there is a linear mapping Ψ of the torus such that

$$\Psi A_1 \Psi^{-1} \in Z(A_2), \Psi P_1 = P_2, (\Theta_2 = \Psi \Theta_1 \Psi^{-1}).$$

and eigenvalues of the matrices $A_2, \Psi A_1 \Psi^{-1}$, corresponding to the same eigenvectors, are identically positioned relative to unit circle of the complex plane.

3. **Arithmetic of integral unimodular matrix and structure of** $Z(A)$.

Theorem C [2]. Let A be a hyperbolic automorphism of the torus defined by an integral matrix A having an irreducible characteristic polynomial and a spectrum of the form

$$SpecA = \{\lambda_1(A), \lambda_2(A), ..., \lambda_s(A), \mu_1(A), \overline{\mu_1}(A), ..., \mu_k(A), \overline{\mu_k}(A)\},$$

where $\lambda_i(A)$ are real and $\mu_j(A)$ are complex. Then the centralizer $Z(A)$ is isomorphic to the group $Z^m \bigoplus F$, where $0 < m < k + s$ and F is finite commutative group.

4. **Questions:** a) Calculating of number m; b) Structure of $Z(A)$, where A is unimodular matrix defined by integral algebraic numbers.

[1] R.V.Plykin, Uspekhi Mat. Nauk 39:6 (1984) , 75-113; English transl., Russian Math. Surveys 39:6 (1984), 85-131

[2] R.V.Plykin, Uspekhi Mat. Nauk 57:6 (2002); English transl., Russian Math. Surveys 57:6 (2002), 1163-1205.