Short wave asymptotics and chaotic solutions

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Studying the propagation of short waves in inhomogeneous media by the application of geometrical optics one arrives at the Hamilton–Jacobi equation for the phase of the wave and then at the so called transport equation governing the amplitude of the wave. When applying this procedure to a linear hyperbolic system, one obtains as the transport equation a first order equation for a single unknown function which is differentiated along appropriate characteristic directions. We will demonstrate, that under some conditions, these transport equations can have chaotic solutions. It is, they constitute dynamical systems defined on appropriate Banach spaces possessing unstable and dense trajectories in these infinite dimensional Banach spaces. As we show this may happen in two cases:

1. The domain (in the physical space) on which the solutions are defined is unbounded, or
2. There exists a trapped characteristic, i.e. a characteristic which for all (positive) times stays in the region.

The simplest prototype of such a transport equation, having chaotic solutions, was proposed by A. Lasota and M. Ważewska–Czyżewska as a model for blood cells proliferation

$$u_t + xu_x = \lambda \cdot u, \quad \lambda > 1, \quad t \geq 0, x \in [0,1].$$

A.Lasota was also the first, who have noticed that in spite of its linearity it has chaotic solutions [1,2]. Later it was studied intensively by J. Myjak and R. Rudnicki [3,4] The line $x = 0$ is a trapped characteristics for this equation.

We demonstrate that in many cases, the existence of chaotic solutions to the transport equations can imply the occurrence of instability and chaos for the original system. Then we show that this chaos can survive perturbations with nonlinear as well as dissipative terms (see also [5]). In a case of a bounded domain the dissipative terms must be degenerated at the trapped characteristic. Chaos is understood here basically in the sense of Devaney [6].

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