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Ising-Bloch transition on the lattice: an approach based on bifurcation theory

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The parametrically forced complex Ginzburg-Landau equation

$$\partial_t A = (1+i\nu)A - (1+i\beta)|A|^2 A + (1+i\alpha)\partial_x^2 A + \gamma A^*, \tag{1}$$

has been intensively studied during the last few years since the seminal work by Coullet et al. [1]. This equation describes a parametrically forced oscillatory media [2], near the 2:1 resonance. Under certain conditions, the system response entrains to the external forcing admitting two possible phases. Fronts separating these two phases can be of two types: Ising and Bloch. In one dimension, for a forcing amplitude γ small enough, the stationary front (Ising) becomes unstable and a moving front (Bloch) arrises. In the particular case of the variational limit ($\nu = \beta = \alpha = 0$), Bloch fronts do not propagate despite their inherent chiriality [1].

Systems in nature are very often discrete. This discreteness may have many different origins. Some examples of these kind of systems are arrays of coupled oscillators, liquid crystals, spin systems, etc. The discreteness introduces new and interesting features not present in the continuum approximation [3]. In this context, it is interesting to analyse the parametrically forced complex Ginzburg-Landau equation on the lattice

$$\partial_t A_j = (1+i\nu)A_j - (1+i\beta)|A_j|^2 A_j + \kappa(1+i\alpha)(A_{j+1} + A_{j-1} - 2A_j) + \gamma A_j^*. \tag{2}$$

Numerical simulations of this equation show a large variety of front types. Some of them are not present in the continuum case. In particular, we observe two types of bistability between different fronts, and two codimension-two points that organise the parameter space of the system. The bifurcations (both, local and global) connecting different front types can be visualised in an appropriate mapping of the many system's variables onto a cylindrical phase space.

Finally, in this contribution we show that all the different front types (as well as their stability and bifurcations) present in the parametrically forced complex Ginzburg-Landau equation on the lattice can be captured by a normal form consisting of two ordinary differential equations.

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