

The dynamics of vehicular traffic with reaction-time delay and stochasticity of drivers

G. Orosz^{1*}, R. E. Wilson², B. Krauskopf², G. Stépán³

¹ Mathematics Research Institute, University of Exeter, Laver Building, North Park Road, Exeter, EX4 4QE, United Kingdom

² Department of Engineering Mathematics, University of Bristol, Queen's Building, University Walk, Bristol BS8 1TR, United Kingdom

³ Department of Applied Mechanics, Budapest University of Technology and Economics, P.O. Box 91, Budapest H-1521, Hungary

* Electronic Address: g.orosz@exeter.ac.uk

Traffic congestion has been recognized as a serious problem in the road network of many countries. The dynamics of highway traffic is examined by investigating a car-following model which includes the reaction time delay of drivers [1]. Qualitative changes in the dynamics of the corresponding system of delay differential equations are studied analytically and by numerical continuation and numerical simulation.

We have found that the uniform flow solution loses its stability via Hopf bifurcations. This results in oscillations which correspond to travelling waves with different wave numbers propagating against the flow of traffic. Normal form calculations show that, due to the inclusion of the delay, the small-amplitude oscillations are unstable in the vicinity of the Hopf bifurcation points, revealing the possibility of bistable behaviour [2, 3].

The full nonlinear dynamics of the system is explored by following branches of oscillating solutions with numerical continuation techniques, and so regions of bistability and co-existence are determined in parameter space. In such domains, depending on the initial condition, the system either tends to the stable uniform flow or to large-amplitude oscillations corresponding to single or multiple stop-and-go traffic jams [4].

In the latter case stop-fronts and go-fronts are developed corresponding to the entry and exit points of traffic jams. A low-dimensional slow dynamics is manifested by the relative motion of these fronts leading to merging or dispersion of traffic jams.

Stochastic effects are introduced by subjecting one of the driver parameters to a random walk. Numerical simulations show that the front dynamics is robust in the sense that these stochastic effects do not destroy it. However, the time needed for merging or dispersing may change radically in different realizations of the stochastic process. The distribution of the amalgamation time suggests that the motion of fronts may be subject to a 'macroscopic' random walk [5].

The essential role of drivers' reaction time is shown: large enough perturbations, caused by, for example, a truck pulling out of its lane, may be enhanced until stop-and-go traffic jams develop, even when the desired uniform flow is stable. These unwanted events may be avoided by introducing temporary regulations provided via overhead gantries.

-
- [1] G. Orosz, R. E. Wilson, and B. Krauskopf. *Phys. Rev. E*, 70(2):026207, 2004.
 - [2] G. Orosz and G. Stépán. *Journal of Nonlinear Science*, 14(6):505–528, 2004.
 - [3] G. Orosz and G. Stépán. *Proceedings of the Royal Society London A, In Press, Published Online*, 2006.
 - [4] G. Orosz, B. Krauskopf, and R. E. Wilson. *Physica D*, 211(3-4):277–293, 2005.
 - [5] G. Orosz, B. Krauskopf, and R. E. Wilson. *Proceedings of the 6th IFAC Workshop on Time-Delay Systems, L'Aquila, Italy*, 2006.