

**Long term effects of small perturbations in a dynamic duopoly**

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We present deterministic and stochastic dynamics on the production costs of Cournot competitions, based on Nash equilibriums of nonlinear R&D investment strategies to reduce the production costs of the firms at every period of the game. We study the behavior of identical firms with the same and different R&D programs. We show that the boundaries of the recovery region and the out of market region for each firm are dynamically determined by the stable manifolds of special equilibrium points of the dynamics. We show in the deterministic case that small changes in the maximum percentage of reduction of production costs of the R&D programs or small changes in the initial production costs of the firms can lead either one firm or the other firm to be out of the market. In the stochastic case, we show the non-linear effects in the expected profits of the firms.