GEOMETRIC NOISE REDUCTION FOR MULTIVARIATE TIME SERIES

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A noise reduction problem occurs when a sequence of states (time series) of a system governed by a deterministic law is recorded using a measurement process subject to error (measurement noise). Many algorithms have been proposed in order to minimize the loss, due to noise, of information about the behavior of the system.

We address [2] the noise reduction problem for multivariate time series. This case is important in laboratory experiments or real world processes in which the state variables of a multivariate dynamical system can be measured through time. We propose a noise reduction algorithm based, as are many existing algorithms, on best local linear fits for the unknown smooth dynamics. However, while existing algorithms use a least squares approach, ours is based on the statistical theory of measurement error models [1] (regression models wherein, as happens in noise reduction problems, both dependent and independent variables are measured with error).

Another aim of our approach is to recover the long run statistical regularity of the underlying dynamics, rather than to separate the noise and the true signal, as do the existing algorithms. Our results seem to indicate the existence of intrinsic bounds for the exact pointwise prediction of the true dynamics although the exact prediction of the long run behavior of the underlying dynamics might be possible, at least in the low noise limit.

We give empirical evidence of the efficiency of our algorithm in the cleaning of Hénon and Lorenz dynamics corrupted by noise with low and high amplitudes, and for time series ranging from 500 to 50000 data points. In the case of short time series, in terms of distance to the attractor, an up to 80% noise reduction is achieved. In the case of larger data sets, greater reductions (up to 95%) are possible. This allows us to recover fine details of the geometric structure of the attractor. We also prove that our algorithm together with some of the most widely used algorithms can be understood in a common framework: all of them are based on orthogonal projections, with respect to some metric, onto optimal linear subspaces.

[1] W.A. Fuller, "Measurement Error Models". John Wiley and Sons (1987).

[2] M.E. Mera and M. Morán, Chaos 16, 013116 (2006).