# Chaotically Spiking Canards in excitable systems with inertial fast manifolds

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In many slow-fast systems, such as those described by the van der Pol-FitzHugh-Nagumo (vdPFN) model [1], the transition from the excitable steady state attractor, to self-sustained relaxation oscillations takes place through a small amplitude quasi-harmonic limit cycle typical of the supercritical Hopf bifurcation. However, due to the split of the time-scales, the Hopf regime is observable only in a very narrow range of parameters around the bifurcation point. Outside this range both the frequency and the amplitude of the Hopf limit-cycle abruptly jump to those of the relaxation oscillations regime. These sudden changes are known as "canard" explosions [2]. In two dimensional phase-spaces the dynamical interplay between fixed points and limit cycles described above is the only possibility. As long as external forcing is not present, chaotic behavior, for example, can be ruled out. However, higher-dimensional relaxation oscillators allow for a more varied and complex dynamics including chaos, mixed mode oscillations, erratic bursting, etc [3].

Motivated by a physical problem, namely the analysis of a Fabry-Perot cavity with one pendulum mirror, we were led to consider a three-dimensional extension of the vdPFN model where the motion on the fast manifold includes inertial terms, thus becoming, by necessity, two-dimensional. The competition between the effects of radiation-pressure and the thermal expansion of the mirrors [4], together with the fact that their intrinsic evolution occurs on different time-scales, provide the basic ingredients for excitable vdPFN dynamics while the mechanical reaction of the pendulum mirror is responsible for the inertial features of the fast motion.

In this generalized vdPFN model, the transition from excitability to relaxation oscillations implies a much more interesting dynamical scenario, where the usual Hopf bifurcation is now followed by a period doubling route to a small-amplitude chaotic attractor. While on the chaotic attractor, the system reacts to external perturbations in an excitable way. Further increasing the control parameter, an intermediate chaotic canard regime then follows in which the small chaotic background eventually triggers excitable spikes in an erratic but completely deterministic sequence, until the relaxation oscillations regime is eventually reached. We finally show that this behavior is consistent with the dynamics observed in a detailed physical model of the pendular cavity.

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