Chaos on Second Order Piecewise Linear Systems

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The chaotic behaviour is present in very low-dimensional non linear dynamical systems. It is known that basic non linear models exhibit dynamical complexity, for example, the Lorenz and Rössler system equations; on the other hand, the jerk type equations are also simple representations of chaotic dissipative flows. Nevertheless, these systems are three dimensional ones. In the present work we show the appearance of chaotic behaviour, in a two dimensional piecewise linear system. The system bears binary hysteresis and its phase space is defined by two surfaces where the system oscillates (slow motion). By means of a switch mechanism (fast motion) the trajectories move from one surface to the other one. The Poincaré-Bendixon theorem requires, besides non linearity, at least three autonomous first-order ordinary differential equations (ODE ' s) to describe the chaotic motion. This apparent violation of the theorem is due to a discontinuity in the variables provided by the switching mechanism at two singular points. This jump mechanism, which is responsible of a stochastic change in the motion, is of great importance in the appearance of chaos. The simplest realization of the dynamical system is an electrical circuit in which we implement two ODE ' s (equivalent to a second order differential equation), and designate as system A. On the other hand we implement a three dimensional one, equivalent to a third order differential equation but with a small highest order coefficient, which we call system B. Both systems, A and B, generate the same chaotic attractor family and therefore the system A derives of the system B when they are compared. Due to this close relation between systems A and B , we can demonstrate, both analytically and numerically, that the expansion and the contraction rates of the trajectories are of the same order of magnitude in both systems. These dynamical systems have a fast jump response which is justified by the great contraction rates obtained in our analysis. Therefore we suggest that this chaos mechanism can be generalized to other physical systems governed by two dimensional piecewise linear systems.