Acceleration and deceleration of particles in billiards with time–dependent boundaries

Alexander Loskutov[1]*, Alexei Ryabov[2]

[1] Physics Faculty, Moscow State University, Moscow 119992, Russia
[2] Physics Department, University of Potsdam, Germany
* Electronic Address: Loskutov@chaos.phys.msu.ru

Billiards are very convenient models of several physical systems. For example, particle trajectories in billiards of specific configuration can be used in modeling a lot of dynamical systems. Moreover, most approaches to the problems of mixing in many-particle systems go back to billiard-like questions. A natural physical generalization of a billiard system is a billiard whose boundary is not fixed, but varies by a certain law. This is a quite new field which opens up prospects in studies of problems that have been known for a long time.

We study the problem of acceleration of particles (Fermi acceleration) in a two-dimensional Lorentz gas with time-dependent scatterer boundaries and in stadium-like billiards with a periodically perturbed boundary. As is known, the usual Lorentz gas (with a fixed boundary) has strong chaotic properties (mixing, decay of correlations, etc.). Perturbations of the boundaries in such a billiard lead to the Fermi acceleration. It is found that the acceleration is higher in the case of periodic boundary oscillations [1].

The results described can be applied to an arbitrary billiard in which a distribution of angle (between the normal to the boundary at a collision point and the particle velocity) is known. Therefore, the technique developed can be used in solving the problem of Fermi acceleration in a general case.

Analysis of stadium-like billiards shows that, for the case of the developed chaos (when this billiard is a classical stadium), the dependence of the particle velocity on the number of collisions has the same character as in the Lorentz gas. At the same time, for a nearly-rectangle stadium a new interesting phenomena is observed. Depending on the initial values, the particle ensemble can be accelerated, or its velocity can decrease up to quite a low magnitude [2]. However, if the initial values do not belong to a chaotic layer then for sufficiently high velocities the particle acceleration is not observed.

On the basis of our investigations we can advance the following conjecture: chaotic dynamics of a billiard with a fixed boundary is a sufficient condition for the Fermi acceleration in the system when a boundary perturbation is introduced. This conjecture is supported by several investigations (see, e.g., [3]).

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