On the problem of topological classification of geometrical Lorenz attractors

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In the report there are considered geometrical properties of attractors which are suspensions of expanding discontinuous map of interval.

Consider the model of Lorenz attractor suggested by R. Williams. Let a branched manifold L and a semiflow ϕ_t, $t > 0$ on L are the result of factorization of a neighborhood of the Lorenz attractor by the one-dimensional invariant stable foliation transversal to the trajectories of the Lorenz flow. Consider the inverse limit

$$\widehat{L} = \lim_{\leftarrow}(L, \phi_t, t > 0).$$

The elements $\widehat{z} \in \widehat{L}$ are the branches of the negative semitrajectories of the semiflow ϕ_t. A flow $\widehat{\phi}_t$ is naturally defined on the space $\widehat{L}$. The pair $(\widehat{L}, \widehat{\phi}_t)$ is called *the geometrical Lorenz attractor.*

We present a topological invariant (*Lorenz manuscript*) leading to the existence of uncountable set of non-homeomorphic geometrical Lorenz attractors [1, 2]. Let $\widehat{O}$ be the fixed point of the flow $\widehat{\phi}_t$, $\widehat{W}^u(\widehat{O})$ be the unstable manifold of the point $\widehat{O}$, $A = clos(\widehat{W}^u(\widehat{O}))\backslash\widehat{W}^u(\widehat{O})$. A symbolic prehistory $\alpha(\widehat{z})$ is a symbolic sequence $\{\alpha_i\}$ characterizing the behavior of the negative semitrajectory $\widehat{z}$. Let Ω be the set of symbolic prehistories of points of A. We call two prehistories $\alpha(\widehat{z}) = \{\alpha_i\}_{i\geq 1}$ and $\alpha(\widehat{w}) = \{\gamma_i\}_{i\geq 1}$ to be equivalent if their "tails" coincide, that is $\exists k, l$ such that for any $m \geq 0$ $\alpha_{k+m} = \gamma_{l+m}$.

Denote the set of equivalence classes by $\widehat{\Omega}$. For each class $\beta \in \widehat{\Omega}$ consider the set $C(\beta) = \{\widehat{z}|\alpha(\widehat{z}) \in \beta\}$.

We call the set of cardinals

$$\chi = \{card(C(\beta))|\beta \in \widehat{\Omega}\}$$

by Lorenz-manuscript.

Theorem[1, 2].

1.χ is a topological invariant, that is if $\widehat{L}_1$ is homeomorphic to $\widehat{L}_2$, then $\chi_1 = \chi_2$;

2. for any sequence $\{n_i\}$ of natural numbers there exists a pair $(\widehat{L}, \widehat{\phi}_t)$ such that $\chi = \{n_1, n_2, ..., \aleph_0, c\}$.

The geometrical Lorenz attractors (Williams' model) and Lorenz type attractors (model of Afraimovich, Bykov and Shil'nikov) are suspensions of expanding map of interval with one point of discontinuity. We construct new examples of attractors, which are suspensions of an expanding map of interval with n points of discontinuity. We consider the geometrical structure of these objects and propose series of topological invariants, constructed by the similar way as the Lorenz manuscript. We prove that all various topological types of these attractors can be obtained by small C^1 perturbation of map of interval.

[1] N.E. Klinshpont, "On the problem of topological classification of Lorenz' type attractors", *Math. Sbornik.* **197**:4 (2006), 75-122.

[2] N.E. Klinshpont, E.A. Sataev and R.V. Plykin, "Geometrical and dynamical properties of Lorenz type system",*Journal of Physics: Conference Series.* **23** (2005), 96-104.