Hyper-acceleration in a stochastic Fermi-Ulam model

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In 1949 Fermi [1] proposed an acceleration mechanism of cosmic ray particles through interaction with a time dependent magnetic field. Ever since, several modifications of the original model have been suggested, one of which is the well-known Fermi-Ulam model (FUM) [2, 3] which describes the bouncing of a ball between an oscillating and a fixed wall. A standard simplification [3] widely used in the literature, the *static wall* approximation (SWA), ignores the displacement of the moving wall but keeps the time dependence in the momentum exchange between particle and wall at the instant of collision as if the wall were oscillating. The SWA speeds up time-consuming numerical simulations and allows semi-analytical treatments as well as a deeper understanding of the system [3, 4]. However, as shown by Einstein in his treatment of the Brownian random walk [6], considering the full phase space trajectory (instead of the momentum component only) is essential for the correct description of diffusion processes. The present work [7] shows, through the investigation of the evolution of an ensemble of particles in a generalized FUM with two randomly oscillating walls, that if the position of the scatterer is neglected, then Fermi acceleration is considerably underestimated. A new simplification is introduced, termed *hopping wall* approximation (HWA). It is shown that HWA can accurately describe the diffusion in momentum space, while at the same time allows analytical treatment and speeds up numerical simulations. By means of this approximation, the root mean square particle velocity V_{rms} is analytically determined and the probability distribution function of the magnitude of particle velocity as a function of the number of collisions n is derived, showing complete agreement with corresponding numerical results for large n. Finally, it is also shown that the effect of hyper-acceleration (amplified acceleration due to scatterer displacement) is also present in higher-dimensional scattering systems such as the driven Lorentz-gas[8]

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