Abstract for Dynamics Days Crete 2006
Topic: Turbulence and Fluid Dynamics

Shear-induced Chaos in Complex Fluid Flow

Chris Goddard[1*], Ortwin Hess[1], Siegfried Hess[2]

[1] Advanced Technology Institute, University of Surrey, UK
[2] Institut für Theoretische Physik, TU-Berlin, Germany
[*] Electronic Address: `c.goddard@surrey.ac.uk`

Due to their intricate micro-structure, complex fluids such as polymers, colloids and micelles can show a variety of nonlinear responses to imposed strain. As a result, the viscosity (which in Newtonian fluids is only a function of temperature and pressure), becomes also a function of the applied strain due to structural rearrangements. Shear thickening or shear thinning (observed by a increase or decrease of viscosity with strain) are common responses, but there also exists a regime where the shear stress in a fluid responds periodically or even chaotically to constant strain forces.

We study a phenomenological Maxwell model [1, 2] for the evolution of the stress tensor in complex fluids for simple plane Couette flow to investigate the nonlinear processes at work. Derived from irreversible thermodynamics or a Fokker-Planck equation, the friction stress tensor $\underline{\underline{\pi}}$ is assumed to obey the following relaxation equation,

$$\frac{\partial}{\partial t}\underline{\underline{\pi}} - 2\overleftrightarrow{\underline{\omega} \times \underline{\underline{\pi}}} - 2\kappa\overleftrightarrow{\underline{\underline{\gamma}} \cdot \underline{\underline{\pi}}} + \tau_0^{-1}(\underline{\underline{\Phi}} - \ell_0^2 \Delta \underline{\underline{\pi}}) = \sqrt{2}\underline{\underline{\gamma}}\,, \qquad (1)$$

where $\underline{\omega}$ is the vorticity, $\underline{\underline{\gamma}}$ is the strain rate, τ_0 is a relaxation time coefficient, ℓ_0 is a characteristic length, $\underline{\underline{\Phi}}$ is the derivative of a potential function and κ is a model parameter. $\overleftrightarrow{\underline{\underline{\cdots}}}$ corresponds to the symmetric traceless part of a tensor.

By varying the parameters in the potential term, we are able to investigate the effect of different temperatures/densities, whilst also having control over the imposed shear rates. For a spatially homogeneous system we show that indeed there are not only periodic responses but also chaos arising from the steady plane Couette shearing flow. The model allows for detailed analysis of the nonlinear processes at work, by using manifold and Floquet stability analysis to classify the behaviour, and using the largest Lyapunov to map the locations of chaotic responses.

Periodically oscillating the plane Couette flow provides a dynamic forcing term for the imposed strain rate. The response of the shear stress from small amplitude oscillatory shearing (SAOS) can be used for measurements of storage and loss moduli through looking at the phase lags. When using large amplitude oscillatory shearing (LAOS) it is possible to further study the nonlinear behaviour by also investigating the frequency dependencies through Fourier transforms [3]. Such techniques are used widely for the classification of polymers where normal methods fail to distinguish between different chemical compositions.

Using the spatially resolved inhomogeneous from of the model under a constant strain we can also show spatiotemporal chaos. Such behaviour could be utilised for potential mixing applications, whereby the 'self mixing' properties combine with the excellent phase space exploration that a chaotic system possesses.

---

[1] Ortwin Hess and Siegfried Hess, *Physica A* **207**, 517-540 (1994).

[2] Ortwin Hess, Chris Goddard and Siegfried Hess, *Physica A* **366**, 31-54 (2006).

[3] Manfred Wilhelm, *Macromol. Mater. Eng.* **287**, 83-105 (2002).