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Systems with different time scales: scaling & reducing of fast chaos, modeling by noise

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We investigate time-continuous dynamical systems which are subjected to a change of the time parameterization.

As first step we analyze the scaling of dynamical properties of a flow of a differential equation under smooth transformation of time

$$\epsilon(x)\frac{dx}{d\tau} = \frac{dx}{dt} = f(x). \tag{1}$$

We give a complete summary of the transformation laws of the main dynamical quantities of interest (e.g. Lyapunov exponents, entropies, fractal dimensions). The results are expected to impact our general understanding in particular of relativistic chaotic dynamics (joint work with A. E. Motter).

As second step we investigate systems of (coupled) differential equations which show a pronounced separation of time scales, due to a small multiplicative factor

$$\begin{aligned} \epsilon\frac{dx}{d\tau} &= f(x,y) \\ \frac{dy}{d\tau} &= g(x,y). \end{aligned}$$

We derive an description of the effective slow dynamics by reducing the fast (chaotic) degrees of freedom. A usual ad-hoc perturbative approach involving a Markov approximation yields to a Fokker Planck equation in the slow variables, with drift and diffusion terms determined by the fast dynamics. Whereas a Young measure approach recovers the first order (drift) terms of such an equation in some cases. The perturbative approach can be applied to Hamiltonian systems with slow and fast chaotic degrees of freedom. The interesting features of the obtained approximation in view of the Hamiltonian nature of the full system are the appearance of damping terms and of the multiplicative nature of the noise which together reflect the energy conservation of the full system. A comparison of the numerical and analytical analysis supports the approach.