Influence of variability and multiplicative noise on a net of FitzHugh-Nagumo elements

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In the last decades it was shown that noise has a strong influence on the dynamics of many nonlinear systems. Well known phenomenona are for instance *stochastic resonance*[1], *noise induced phase transitions*[2].

The system under consideration is a network of $N \times N$ coupled FitzHugh-Nagumo (FHN) [4] elements. A special noise induced phase transition in a net of FHN elements from oscillatory to excitable behaviour due to white respectively colored multiplicative noise was reported in [3, 4]. Beside parametric noise we examine the influence of variability, which denotes static stochastic differences between the otherwise equal elements of the net, on the dynamics of the network. Variability is - like noise - omnipresent in nature and means here that a parameter differs from element to element, gaussian distributed around its forced mean. The model equations with noise and variability in several parameters read:

$$\begin{aligned} \frac{du_{ij}}{dt} &= \frac{1}{\epsilon}(u_{ij}(1-u_{ij})(u_{ij}-a_{ij}(1+\xi_{ij}(t))) - v_{ij} + d) + D_u \nabla^2 u_{ij} \\ \frac{dv_{ij}}{dt} &= b(u_{ij} - c_{ij}(1+\eta_{ij}(t))v_{ij}) + e_{ij} + \mu_{ij}(t)\,, \end{aligned} \tag{1}$$

with variability in the parameters a, c and e and where $\xi_{ij}(t)$, $\eta_{ij}(t)$ and $\mu_{ij}(t)$ represent temporally colored spatially white noise fields. In [5] was shown that variability in parameter c has evidently a very similar influence on the global network dynamics as multiplicative noise in parameter c. Analog studies to the influence of noise and variability in parameters a and e are done. Modifying parameter a via noise respectively variability leads again in both cases to a quite similar phase transition.

Variability in parameter e is the companion piece to the additive noise term $\mu_{ij}(t)$ and can also evoke pattern formation (spiral waves) in an excitable net of FHN elements. Adding a weak cos-signal on the variable $v_{ij}(t)$ variability in parameter e can amplify this external signal. The linear response of the net of FHN elements to this signal shows a clear maximum for intermediate strength of variability comparable to stochastic resonance, where an optimal amplification of a signal is reached for intermediate noise strength.

We are going on to investigate the influence of noise and variability on a net of FHN elements to get the idea why noise and variability act so similar on the network dynamics. the deterministically influence of variability.

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