Global Analysis of Planar Polynomial Dynamical Systems

V. A. Gaiko*

Department of Mathematics
Belarusian State University of Informatics and Radioelectronics
L. Beda Str. 6–4, Minsk 220040, Belarus
* Electronic Address: vlrgk@yahoo.com

Two-dimensional polynomial dynamical systems are considered. The main problem of the qualitative theory of such systems is Hilbert's Sixteenth Problem on the maximum number and relative position of limit cycles. There are three well-known local bifurcations of limit cycles: 1) Andronov–Hopf bifurcation (from a singular point of center or focus type); 2) separatrix cycle bifurcation (from a homoclinic or heteroclinic orbit); 3) multiple limit cycle bifurcation (from a multiple limit cycle of even or odd multiplicity). We connect all these local bifurcations by means of the Wintner–Perko termination principle and develop new global approaches to solving the Problem [1]. First, applying Erugin's two-isocline method, we construct a quadratic canonical system with four field-rotation parameters and, using geometric properties of these parameters, prove by two ways that a quadratic system has at most three limit cycles around a singular point (focus) confirming our conjecture that the maximum number of limit cycles in a quadratic system is equal to four and the only possible their distribution is $(3:1)$ [1]. By means of Erugin's method, we construct also a canonical Kukles-type cubic system with field-rotation parameters and apply it for studying limit cycle bifurcations. In particular, we consider a special case of the Kukles system which corresponds to a generalized Liénard equation and is very important for applications, classify its separatrix cycles and study global bifurcations of limit cycles [2]. Finally, we study a general Liénard system of arbitrary degree solving the problem on the maximum number of its limit cycles [3] and establish the global qualitative analysis of a cubic centrally symmetric dynamical system which can be used as a learning model of planar neural networks [4].

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