Synchronization of chaotic oscillators as a phase transition

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We characterise the synchronization of an array of coupled chaotic elements as a phase transition where order parameters obey power laws versus the coupling strength. Since however we intend to model a task, whose completion occurs within a limited time slot, we focus on the transient behaviour of the transition. Assuming that a task is performed when all elements are synchronized, we investigate the dynamics of the defects, which mark the boundary between different synchronization domains, finding the coupling strength above which the initial defects decay within an assigned time.

Temporal coding vs. rate coding for the neural-based information has been open to debate in the neuroscience literature[1]. Experimental evidence of the electrical activity of a single neuron is obtained by insertion in the cortical tissue of animals of microelectrodes each applied to a single axon[2, 3]. This activity consists of trains of action potentials or 'spikes'. In rate coding only the mean frequency of spikes over a time interval matters, thus requiring a suitable counting interval, which seems unfit for fast decision tasks. Temporal coding assigns importance to the precise timing of spikes. A special type of temporal coding is synchrony, whereby information is encoded by the synchronous firing of spikes of all neurons of a cortical module.

Let an animal be exposed to a visual field containing two separate objects, both made of the same visual elements. Since each receptive field isolates a specific detail of an object, we should expect a corresponding large set of different responses. On the contrary, all the cortical neurons whose receptive fields are pointing to a specific object synchronize their spikes, and as a consequence the visual cortex organizes into separate neuron groups oscillating on distinct spike trains for different objects (feature binding).

The dynamics ruling the above facts should be described by a convenient model, independently of the biological components. Since feature binding results from the readjustment of the temporal positions of the spikes, a plausible explanation is based on the mutual synchronization of chaotic oscillators; a chaotic distribution of the spike times seems mandatory for adapting to different rhythms ([4], and references therein).

In our study we explore the conditions under which an assembly of coupled chaotic oscillators reaches mutual synchronization, thus displaying a coherent behavior. We compare two dynamical models, namely HC[4] and Rössler[5].

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[1] W. Gerstner and W.M. Kistler, *Spiking Neuron Models. Single Neurons, Populations, Plasticity*, Cambridge University Press, 2002 and references therein.

[2] W. Singer and E.C.M. Gray, *Annu. Rev. Neurosci.* **18**, 555 (1995); G.N. Borisyuk et al, *Physics-Uspekhi* **45**, 1073 (2002).

[3] S. Grossberg, *Am. Sci.* **83**, 439 (1995).

[4] F.T. Arecchi, *Physica A* **338**, 218-237 (2004).

[5] O.E. Rössler, *Phys. Lett. A* **57**, 397 (1976).