Stability and Chaos in Multi - Degree of Freedom Hamiltonian Systems

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Some aspects of the problem of the global stability of motion of N - degree of freedom Hamiltonian systems will be discussed. Starting with N small, I shall proceed to the case of N arbitrarily large, in an attempt to understand the thermodynamic limit, where N is large and statistical mechanics is expected to take over from classical mechanics. Of primary importance in this discussion is the connection between local and global dynamics, i.e. the relevance of events occurring in small - scale regions of the energy surface to the stability of motion in large domains. This link is provided by a study of I call Simple Periodic Orbits (SPOs), i.e. periodic solutions with all degrees of freedom oscillating in phase (or out of phase) and with equal characteristic frequencies. Applying Lyapunovs theory of the continuation of normal modes of N - degree of freedom Hamiltonian systems to the famous Fermi - Pasta - Ulam (FPU) lattice, we shall explain how these SPOs help us resolve the paradox of the FPU recurrences. I will discuss how the study of SPOs in the FPU and other Hamiltonians, can shed new light on the transition to large scale chaotic behavior characterized by invariant spectra of Lyapunov exponents. Finally, I will report on a recent discovery of a very efficient spectrum of indices distinguishing ordered from chaotic motions in conservative dynamical systems, called the GALIk ,k=1,2,..,2N, which apply not only to Hamiltonian systems, but also to 2N - dimensional symplectic maps.

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[1] C. Antonopoulos, T. Bountis and Ch. Skokos, "Chaotic Dynamics of N - Degree of Freedom Hamiltonian Systems", Int. Journal Bifurc. Chaos, to appear, June 2006.

[2] C. Antonopoulos and T. Bountis , "Stability of Simple Periodic Orbits and Chaos in an FPU Lattice", PRE, to appear, June 2006.

[3] T. Bountis, "Stability of Motion: From Lyapunov to N - Degree of Freedom Hamiltonian Systems", Proceedings of "Lets Face Chaos Through Nonlinear Dynamics", Maribor, Slovenia, June 26 - July 10, 2005, to appear in Progr. Theor. Phys. 2006.