Modulated point vortex couples on a β-plane: dynamics and chaotic advection

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The presence of gradients in the background vorticity makes even the dynamics of point vortices surprisingly nontrivial, and the advection to be chaotic. In geophysical flows such a gradient naturally appears due to the so called "beta-effect".

Large scale fluid flows on Earth are basically influenced by the Coriolis force, and the global dynamics is determined by the conservation of potential vorticity. The Coriolis parameter varies in the latitudinal direction. In the "β-plane" approximation this variation is represented by a term depending linearly on the latitudinal coordinate y. In the simplest case of a homogeneous, ideal fluid of fixed depth, the potential vorticity $q \equiv \zeta + \beta' y$ is a conserved quantity in the course of the fluid motion, where ζ is the local vorticity and β' represents the beta parameter.

The dynamics of such modulated point vortex couples on a β-plane is investigated for arbitrary ratios of the vortex strength. To analyze the motion we propose a new potential depending on the longitudinal coordinate and show that qualitative changes of the vortex orbits follow as bifurcation-like events in the shape of this potential. In certain ranges of parameters the potential has the unusual feature of being bi-valued. The motion along the two branches of the potential result in smooth trajectories, but the change from one branch to the other leads to a point of non-differentiability. In this way surprizing orbits with spikes appear in the dynamics in spite of the integrability of the problem.

The advection dynamics in this modulated two-vortex problem is chaotic. We point out a transition from closed to open chaotic advection implying that the transport properties of the flow might drastically be altered by changing some parameters or the initial conditions. The open case, characterized by a permanent entrainment and detrainement of particles around the vortices, is interpreted in terms of an invariant chaotic saddle of the Lagrangian dynamics, while the dynamics of the closed case, with a permanently trapped area of the fluid, is governed by a chaotic region and interwoven KAM tori.