

Lyapunov exponents calculation for distributed nonlinear resonators

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We present the results of numerical calculations of quantitative characteristics of complex regimes in some models of distributed systems. We have developed a code to calculate Lyapunov exponents spectra in such systems, based on a modification of Benettine algorithm. To check the preciseness of our programm we have tested it in application to systems with time delay studied in [1].

In this paper we discuss the investigation of complex dynamics in nonlinear resonators filled by media with modulation instability driven by an external harmonic signal. We examined two models: the model of ring cavity, described by nonlinear Schrödinger equation (NLS) with delay term, and the model of 1-D resonator (the piece of nonlinear medium with end reflections), described by a system of coupled NLS.

Previously we have carried out the computer modeling of those systems in a wide range of parameters [2]. Here we present the results of numerical simulation of different regimes with corresponding Lyapunov exponents (LE) spectra. We observed regimes of periodic oscillations (with all exponents being negative), quasi-periodic motion (with zero LE), chaos (one positive LE) and hyperchaos (the case when system possesses more than one positive Lyapunov exponents). The last phenomenon had been already observed in numerical simulation of backward wave oscillator [1] and in gyrotron with non-fixed longitudinal field structure [3]. We also found, that in some hyperchaotic regimes Kaplan-Yorke dimension of the attractor is extremely high. The last effect had been found out in gyrotron in [3].

We also examined the cases of weak dispersion when discussed models can be reduced to Ikeda map (NLS with a delay term) or to the system of coupled Ikeda maps (model of 1-D resonator with end reflections). We present the results of comparison of Lyapunov exponents calculated for both distributed and reduced systems and found good coincidence for small input signal intensities.

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