Lyapunov Spectra and Hierarchies of Chaos and Order in Multi – Degree of Freedom Hamiltonian Systems

Chris Antonopoulos*

Department of Mathematics and Center for Research and Applications of Nonlinear Systems (CRANS), University of Patras, Patras, GR – 26500, Greece
* Electronic Address: antonop@math.upatras.gr

In this work, we discuss "hierarchies" of chaotic and ordered motion of certain examples of N degree of freedom Hamiltonian systems, which include the famous Fermi – Pasta – Ulam (FPU) lattice and a discretized version of the nonlinear Schröedinger equation related to the Bose – Einstein Condensation. We study solutions in the vicinity of simple periodic orbits (SPOs) of these systems corresponding either to continuations of their linear normal modes, or in – phase and out – of – phase motions, which are known in closed form and whose linear stability can be analyzed exactly. We find that chaotic regions about such unstable SPOs can be quite distinct when they are characterized by different Lyapunov spectra, and "merge" together, when their spectra converge. Since our results hold for N arbitrarily large, they suggest a direct approach by which one can use stability analysis of SPOs and Lyapunov spectra to estimate energy thresholds where a transition to ergodicity occurs and such thermodynamic properties as Kolmogorov – Sinai entropies can be defined. Finally, applying the recently introduced Generalized Alignment index GALI_k, as well as the Linear Dependence Indicator LDI_k on these Hamiltonians, one not only distinguishes rapidly and efficiently between regular and chaotic motion, but also discovers different "hierarchies" of order, as there are cases of the FPU system, where regular dynamics occurs systematically on tori whose dimensionality is lower than N!

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