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Topic: Quantum Chaos

Quantum ratchet in a fractional kicked rotor

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Quantum dynamics of a fractional kicked rotor is considered. The Hamiltonian of the system is

$$\hat{H} = \hat{\mathcal{T}} + \epsilon \cos x \sum_{n=-\infty}^{\infty} \delta(t-n) \,, \tag{1}$$

where ϵ is an amplitude of the periodic perturbation. The kinetic part of the Hamiltonian is modeled by the fractional Weyl derivative $\hat{\mathcal{T}} = (-i\tilde{h})^{\alpha}\mathcal{W}^{\alpha}/\alpha$, where $\tilde{h}$ is a dimensionless Planck constant, and $\alpha = 2-\beta$ with $0<\beta<1$. When $\beta = 0$, Eq. (1) corresponds to the quantum kicked rotor. For a periodic function $f(x)$ the Fourier transform determines the fractional Weyl derivative $\mathcal{W}^{\alpha}$ in the following way (see [1], ch. 4.3)

$$\mathcal{W}^{\alpha} f(x) = \sum_{n=-\infty}^{\infty} (-ik)^{\alpha} \bar{f}_k e^{-ikx} \,. \tag{2}$$

Thus, the kinetic term in the Hamiltonian (1) is defined on the basis $|k\rangle = e^{ikx}/\sqrt{2\pi}$ as follows

$$\hat{\mathcal{T}}|k\rangle = \mathcal{T}(k)|k\rangle = \frac{(\tilde{h}k)^{2-\beta}}{2-\beta}|k\rangle \,. \tag{3}$$

This non-Hermitian operator has complex eigenvalues for $k<0$, which are defined on the complex plain with a cut, such that $1^{-\beta} = 1$ and $(-1)^{-\beta} = \cos\beta\pi - i\sin\beta\pi$.

A quantum map for the wave function $\psi(x,t+1) = \hat{U}\psi(x,t)$ with an evolution operator $\hat{U}$ on the period is studied numerically. A specific property of this Hamiltonian dynamics is dissipation resulting in the probability leakage which is described by the survival probability $P(t)$. Another specific characteristic is the non-zero mean value of the orbital momentum $\langle p(t)\rangle$. The first main result is the quantum accelerator dynamics, $\langle p(t)\rangle \sim t^{\gamma_1}$, which is accompanied by the power law decay of the survival probability $P(t) \sim t^{-\gamma_2}$. Quantum localization affects strongly both γ_1 and γ_2. By increase of the quantum effect, when $\tilde{h} = 1$, the exponent γ_1 approaches zero, and, as a result, a quantum ratchet like behavior takes place. The survival probability decays at the rate $\gamma_2 \approx 1$.

The quantum–to classical transition is performed exactly. The classical Green function

$$K_{\tilde{h}=0}(x,p|x'p') = \Theta(p)\delta(x-x'-\omega(p))\delta(p-p'-\epsilon\sin x') \tag{4}$$

corresponds to the classical map of the kicked rotor with the nonlinear frequency $\omega(p) = p^{1-\beta}$ for $p>0$, while the absorbing boundary conditions for $p<0$ are due to the Heaviside function $\Theta(p)$.

The classical–to–quantum transition can be performed exactly, as well. Therefore, the second result is that the fractional Schrödinger equation with the non–Hermitian Hamiltonian (1) is the quantum counterpart of the open system in Eq. (4).

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[1] B.J. West, M. Bologna, and P. Grigolini, " Physics of Fractal Operators", Springer, New York, (2002).